Letters

Comments on "A Simple Numerical Method for the Cutoff Frequency of a Single-Mode Fiber with an Arbitrary Index-Profile"

J. P. MEUNIER, J. PIGEON, AND J. N. MASSOT

In the above paper[1] [1], Sharma and Ghatak have proposed a numerical method for calculating the cutoff frequency of single mode operation in optical fibers with an arbitrary index profile. We want to comment on the validity of their results using their notations. The referenced paper requires in particular the knowledge of the boundary conditions on the modal field Ψ and its derivatives at $R=0$. To this aim, Sharma and Ghatak have used a series solution method. Their series expansion [1, eq. (A.1)] is valid only in the vicinity of an ordinary point but the point $R=0$ is a regular singular point for the scalar wave differential equation [1, eq. (A.3)]. It is well known that in this situation, the correct series expansion is given by [3], [4]

$$\Psi(R)=R^s\sum_{n=0}^{\infty}a_nR^n, \qquad \text{with } a_0\neq 0 \tag{1}$$

where the parameter s is a solution of the indicial equation associated with [1, eq. (A.3)], which can be written in this case as

$$s^2-m^2=0, \qquad m=0,1,2,\cdots \tag{2}$$

providing that the refractive-index profile satisfies the following (physical) condition:

$$\lim_{R\to 0} R^2P(R)=0. \tag{3}$$

In order that the field Ψ must be finite at $R=0$, only the positive root $s=m$ of (2) is physically acceptable.

In addition, we recall that the series expansion (1) always exists if the quantity $Q(R)=R^2P(R)-m^2$ is analytic at $R=0$ and the coefficients a_n are determined by recurrence relations [3], [4]. Therefore, we can obtained in a straightforward way the expressions for Ψ and its derivatives at $R=0$, we get

$$\left.\frac{dp\Psi}{dR^p}\right|_{R=0}=\begin{cases}p!a_{p-m}, & \text{for } m\leqslant p, \quad p=0,1,2,\cdots\\ 0, & \text{for } m>p, \quad p=0,1,2,\cdots\end{cases} \tag{4}$$

Thus, for $m=1$ we have instead of [1, eqs. (A.9) and (A.10)]

$$\Psi(0)=0 \tag{5}$$

$$\left.\frac{d\Psi}{dR}\right|_{R=0}=a_0\neq 0 \text{ and } \left.\frac{d^2\Psi}{dR^2}\right|_{R=0}=2a_1. \tag{6}$$

Our results differ for the value of the second derivative at $R=0$, which is essential to obtain [1, eq. (14)]. It is obvious that for refractive index profiles which lead to $a_1=0$ our results are identical with those obtained by Sharma and Ghatak. Moreover, it can be easily shown that this is precisely the case for the truncated parabolic profile [5] and the step-index profile with a Lorentzian dip, which have been tested by Sharma and Ghatak. But it is certainly not the case, e.g., for an α-profile with $\alpha=1/2$. However that may be, the strict application of the Froebenius method leads to results which are different with those of Sharma and Ghatak so that their method cannot be applied to any arbitrary index profile.

Manuscript received July 16, 1981.

The authors are with the Laboratoire Traitement du Signal et Instrumentation Université de Saint-Etienne, U. E R de Sciences 23, rue du Docteur Paul Michelon, 42023 Saint-Etienne Cédex, France.

[1]A. Sharma and A. K. Ghatak, *IEEE Trans. Microwave Theory Tech*., vol MTT-29, pp. 607–610, June 1981.

Reply[2] by Anurag Sharma and A. K. Ghatak[3]

The boundary conditions at $R=0$ used in our paper[1] [1, eqs. (5), (10), and (14)] are strictly correct for all realistic profiles including the power law profile, for which

$$f(R)=R^{\alpha}, \qquad \alpha>0. \tag{7}$$

In fact, we have carried out numerical calculations (using the method developed in [1]) corresponding to $\alpha=0.50$ and $\alpha=0.25$ and have obtained 5.7343 and 7.7639 as the respective values for the cutoff frequency of single mode operation; for $\alpha=0.25$ Snyder and Sammut [2] have obtained the same value. Indeed the boundary conditions can be derived without assuming any particular form of the solution in a manner shown below.

The wave equation can be written as

$$\Psi''+(\Psi'/R)-m^2(\Psi/R^2)+P(R)\Psi(R)=0 \tag{8}$$

where primes denote differentiation with respect to R and $P(R)=U^2-V\Delta f(R)$, which is finite everywhere. In order that Ψ is finite at $R=0$

$$\lim_{R\to 0}\frac{1}{R}\left[\Psi'-m^2\frac{\Psi}{R}\right] \tag{9}$$

should also be finite. Thus

$$\Psi(0)=0, \qquad m\neq 0. \tag{10}$$

Further, for $m\neq 0$

$$\lim\frac{1}{R}\left[\Psi'-m^2\frac{\Psi}{R}\right]=\lim_{R\to 0}\left[\frac{R\Psi'-m^2\Psi}{R^2}\right]=\frac{1}{2}\Psi''+\frac{(1-m^2)}{2}\frac{\Psi'}{R}. \tag{11}$$

For the above expression to be finite, we must have

$$(1-m^2)\left.\frac{\Psi'}{R}\right|_{R=0}=0 \text{ or } \Psi'_{(0)}=0, \qquad \text{for } m\neq 1. \tag{12}$$

It can be shown that for $m=1$, $\Psi'|_{R=0}=0$ would correspond to the trivial solution ($\Psi(R)=0$) and hence, for a nontrivial solution

$$\Psi'_{(0)}=0, \qquad \text{for} \quad m=1. \tag{13}$$

[2]Manuscript received July 16, 1981.

[3]The authors are with the Department of Physics, Indian Institute of Technology, Delhi, New Delhi-110016, India

0018-9480/82/0100-0108$00.75 ©1982 IEEE

Substituting (11) in (8) and taking the limit $R \to 0$, we get

$$\frac{(4-m^2)\Psi''}{2} + P(0)\Psi(0) = 0 \tag{14}$$

which gives

$$\Psi''(0) = 0, \qquad m \neq 2. \tag{15}$$

Equations (10), (13), and (15) justify all the boundary conditions used in [1]. Thus, the numerical method developed in [1] is valid for an arbitrary index profile including noninteger α-profiles.

We would like to mention that the derivation given in [1, appendix] is strictly valid only for index profiles which are analytic at $R=0$ and it corresponds to the solution which would be obtained by taking the positive root of the indicial equation (2). Indeed, corresponding to the solution $s=m$, we will have $a_0 = a_1 = a_2 = \cdots = a_{m-1} = 0$ [1, eq. (A.8)]. Thus, the considerations put forward by Meunier *et al.* are consistent with [1, appendix]. It should be pointed out that for a power-law profile (7) with α taking noninteger values, the series solutions [1, eq. (A.1)], and (3) are not valid; however, the boundary conditions used in [1] remain valid as shown above.

We would like to take this opportunity to correct few errors in [1, appendix].

df/dR in (A.11) should read dF/dR.

(A.7) should read

$$\sum_{n=0}^{\infty} \left[\{ (n+2)^2 - m^2 \} a_{n+2} + P(R) a_n \right] R^n = 0.$$

(A.8) should read

$$\frac{d^p\Psi}{dR^p} \begin{cases} \neq 0, & p = m \\ = 0, & p < m \text{ and } p = m+1. \end{cases}$$

REFERENCES

[1] Anurag Sharma and A K. Ghatak, "A simple numerical method for the cutoff frequency of a single mode fiber with an arbitrary index-profile," *IEEE Trans. Microwave Theory Tech*, vol. MTT-29, pp. 607–610, June 1981.

[2] A. W. Snyder and R. A. Sammut, "Fundamental (HE_{11}) modes of graded optical fibers," *J. Opt. Soc Amer*, vol. 69, pp. 1663–1671, Dec. 1979.

[3] E. T. Whittaker and G. N Watson, *A Course of Modern Analysis*. New York. Cambridge University Press, 1927, ch. 10.

[4] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, Part I. New York: McGraw Hill, 1953, ch. 5.

[5] W A. Gambling, D. N. Payne, and H Matsumura, "Cutoff frequency in radially inhomogeneous single mode fiber," *Electron. Lett.*, vol. 13, pp. 139–140, Mar. 1977.

Correction to "Exact Analysis of Shielded Microstrip Lines and Bilateral Fin Lines"

A.-M. A. EL-SHERBINY

The following corrections should be made to the above paper.[1]

On page 670, column 2, paragraph 2, the expression "electric wall symmetry (microscope case)" should read "electric wall symmetry (microstrip case)."

On page 672, column 1, the expressions for U_1 and U_2 should read

$$U_1(\alpha) = U_1^-(\alpha) - e^{i\alpha W} U_1^-(-\alpha)$$
$$U_2(\alpha) = U_2^-(\alpha) + e^{i\alpha W} U_2^-(-\alpha).$$

In the same column, (12) should read

$$i\omega\epsilon_0 \chi_1(\alpha) F_1(\alpha) = U_1^-(\alpha) - e^{i\alpha W} U_1^-(-\alpha)$$
$$\frac{1}{i\omega\mu_0} \chi_2(\alpha) F_2(\alpha) = U_2^-(\alpha) + e^{i\alpha W} U_2^-(-\alpha)$$

i.e., in both cases, the signs should be changed.

Manuscript received August 18, 1981.

The author is with the Faculty of Engineering, Ain Shams University, Cairo, Egypt.

[1] A.-M. A. El-Sherbiny, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 669–675, July 1981.

0018-9480/82/0100-0109$00.75 ©1982 IEEE